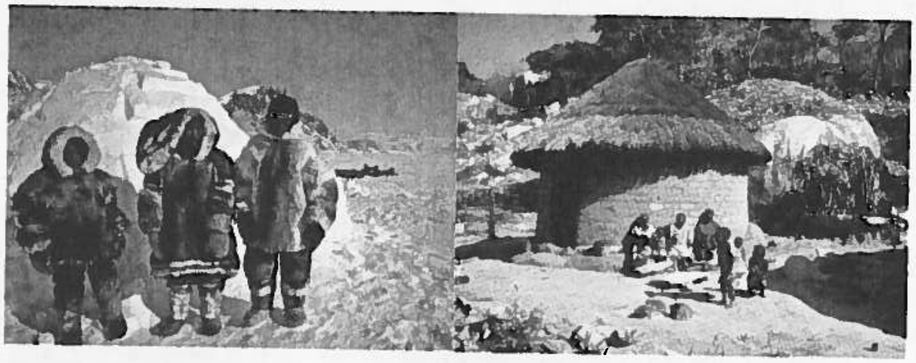
3.5 Properties of Circles

From the Canadian Arctic to Southern Africa, people have developed ingenious circular structures to use as homes or temporary shelters. The materials available and the climate greatly influence the design of these structures.



Investigate

What properties do circles have?

Tools

- circular object such as a juice can
- compasses
- ruler

Method 1: Use Pencil and Paper

1. Draw a circle on a sheet of paper by tracing around a circular object such as a juice can.
2. Find the centre of the circle by folding your drawing. Explain how you know where the centre is.
3. **Reflect** What property do diameters of a circle have?
4. Mark any two points on the circle and label them A and B. Join the points to form a **chord** of the circle. Draw the right bisector of this chord. What property does this right bisector have? Check whether the right bisectors drawn by your classmates have the same property.

:hord
line segment joining two points on a curve

5. Choose any point on a blank sheet of paper, but do not mark this point. Instead, mark three points that are all about the same distance from the first point. Label these three points P, Q, and R. Exchange your set of points with a classmate.
6. Find the centre of the circle that passes through the three points that your classmate marked. Explain your method. Draw the circle that passes through the points.
7. **Reflect** What property do the chords of a circle have?

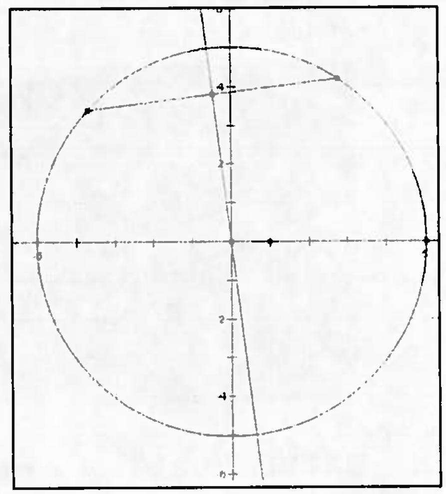
Tools

- computer with *The Geometer's Sketchpad®*

Method 2: Use *The Geometer's Sketchpad®*

1. Turn on the grid display. Construct a point at the centre of the grid and a second point near the edge. Using the first point as the centre, construct a circle by choosing **Circle by Centre and Point** from the **Construct** menu.

2. Construct any two points on the circle. Construct a **chord** by joining these two points with a line segment.



3. Construct the right bisector of the chord.

4. Select one of the endpoints of the chord. From the **Display** menu, choose **Animate Point.** As the endpoint of the chord moves around the circle, watch the right bisector of the chord. What property does this bisector have?

5. **Reflect** What property do the chords of a circle have? How you could use this property to find the centre of a circle given three points on the circumference?

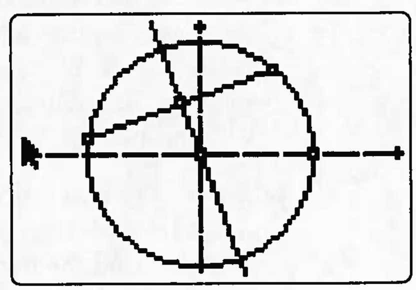
Tools

- TI-83 Plus or TI-84 Plus graphing calculator

Method 3: Use a Graphing Calculator

1. Start the Cabri® Jr. application. Check that the axes are displayed. Choose **Circle** from the **F2** menu. Place the centre near the middle of the screen, and place the end of the radius close to the top of the screen.

2. Choose **Segment** from the **F2** menu. Construct a **chord** by using a line segment to join any two points on the circle.



3. Choose **Perp. Bis.** from the **F3** menu, and construct the right bisector of the chord.

4. Move the cursor to one endpoint of the chord, and press ALPHA. Watch the right bisector of the chord as you move the endpoint around the circle. What property does this bisector have?

5. **Reflect** What property do the chords of a circle have? How could you use this property to find the centre of a circle given three points on the circumference?

Example 2 Points on a Circle

a) Show that the points P(9, −3), Q(8, 6), and R(−1, 5) lie on a circle with its centre at C(4, 1).

b) Does any other circle pass through points P, Q, and R? Explain.

Solution

a) If the three points lie on a circle centred at (4, 1), each point must be the same distance from (4, 1). Compare the lengths of CP, CQ, and CR.

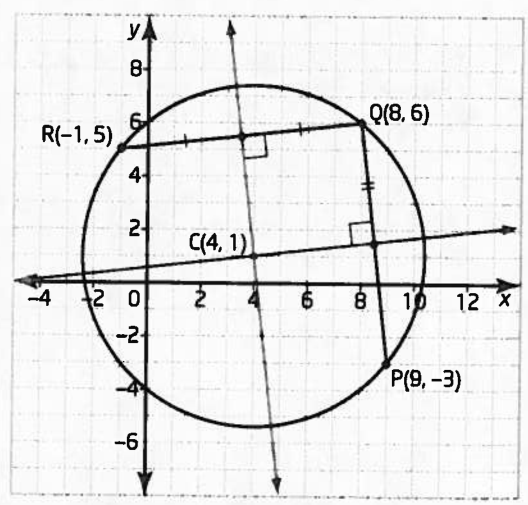
$$\begin{aligned} CP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(9 - 4)^2 + (-3 - 1)^2} \\ &= \sqrt{5^2 + (-4)^2} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} CQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 4)^2 + (6 - 1)^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} CR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 4)^2 + (5 - 1)^2} \\ &= \sqrt{(-5)^2 + 4^2} \\ &= \sqrt{41} \end{aligned}$$

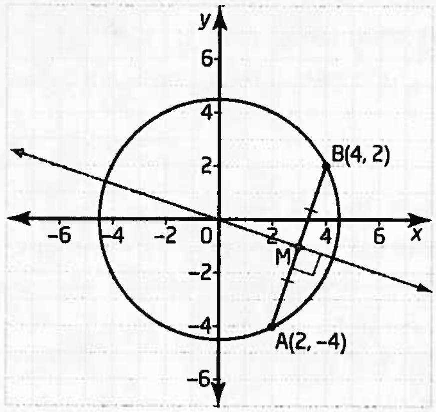
Since CP = CQ = CR, the points P, Q, and R all lie on a circle centred at C(4, 1).

b) The right bisector of PQ includes all points that are equidistant from P and Q. Similarly, the right bisector of QR includes all points that are equidistant from Q and R. These two lines meet only at point C(4, 1). There is no other point equidistant from P, Q, and R.



Example 1 Right Bisector of a Chord

Verify that the centre of this circle lies on the right bisector of the chord AB.



Solution

Understand the Problem

The centre lies on the right bisector of AB only if the coordinates (0, 0) satisfy the equation of the right bisector.

Choose a Strategy

The midpoint of AB is on the right bisector. Use the coordinates of A and B to find the midpoint and the slope of AB. Then, calculate the slope of a line perpendicular to AB. Use this slope and the coordinates of the midpoint, M, to determine the equation of the right bisector of AB.

Carry Out the Strategy

Find the midpoint coordinates and the slope of AB.

$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2 + 4}{2}, \frac{-4 + 2}{2}\right) \\ &= (3, -1) \end{aligned}$$

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-4)}{4 - 2} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

Since the right bisector is perpendicular to AB, the slope of this bisector is the negative reciprocal of m_{AB}.

$$-\frac{1}{m_{AB}} = -\frac{1}{3}$$

Use this slope and the coordinates M(3, −1) to find the y-intercept.

$$\begin{aligned} y &= mx + b \\ -1 &= -\frac{1}{3}(3) + b \\ -1 &= -1 + b \\ 0 &= b \end{aligned}$$

All lines with a y-intercept of 0 pass through the origin.

Reflect

The equation of the right bisector is $y = -\frac{1}{3}x$. The coordinates (0, 0) satisfy this equation. Therefore, the centre of the circle lies on the right bisector of the chord PQ.